Channel Estimation in mmWave Hybrid MIMO System via Off-Grid Dirichlet Kernels

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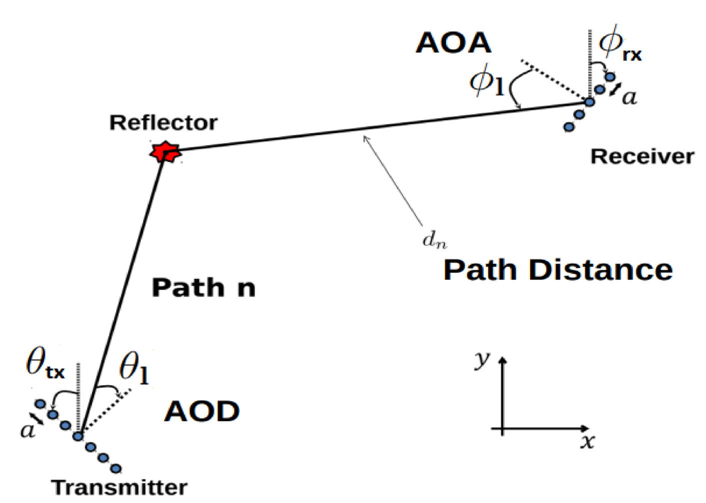
General Overview

- We consider the channel estimation problem for a mmWave MIMO along with hybrid beamforming and basis mismatch (i.e., off-grid).
- Compressed sensing methods do not leverage the **Dirichlet structure** in the Fourier domain.
- Orthogonal matching pursuit (OMP) based estimation algorithm exploiting Dirichlet structure is proposed.

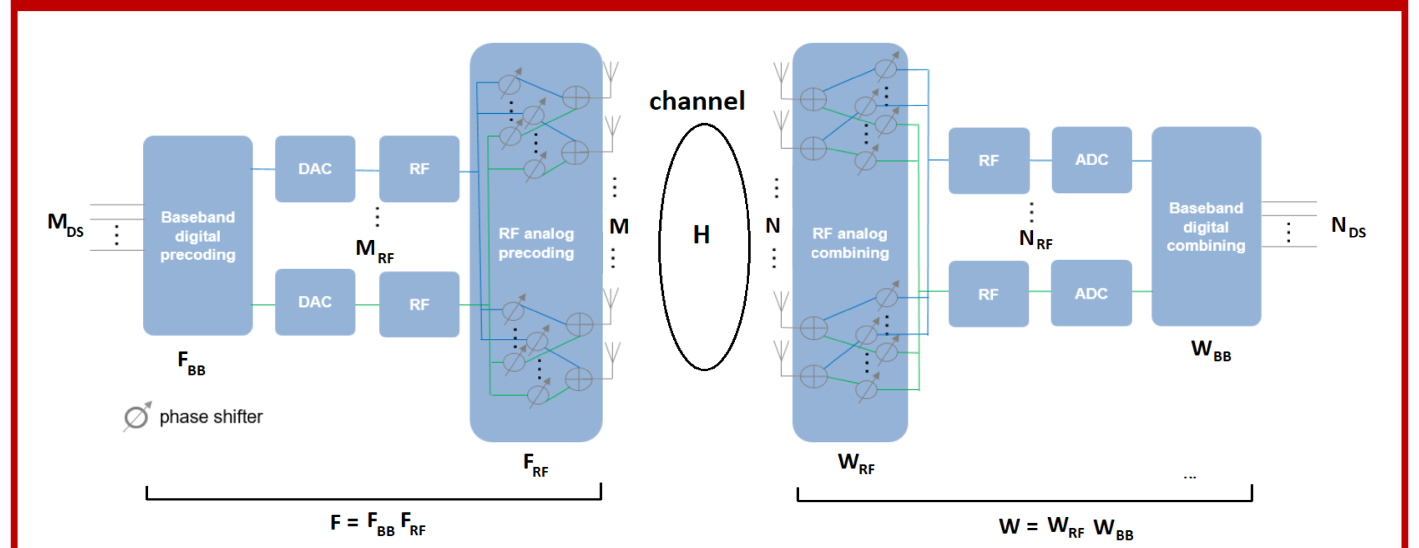
Channel Model



$$\mathbf{H} = \sum_{l=1}^{L} \alpha_l \mathbf{a}_{\mathbf{UE}}(\theta_l)\mathbf{a}_{\mathbf{BS}}{}^{\mathbf{H}}(\phi_l)$$

- $\alpha_l \rightarrow$ Complex gain of l^{th} MPC
- $\theta_l/\phi_l \rightarrow$ AoA/AoD of l^{th} MPC
- $\mathbf{L} \rightarrow$ Number of MPCs
- $\mathbf{a}_{\mathbf{UE}}/\mathbf{a}_{\mathbf{BS}} \rightarrow$ UE/BS array response
- MPC $\rightarrow$ Multi-path component
- UE $\rightarrow$ User equipment
- BS $\rightarrow$ Base station
- AoA $\rightarrow$ Angle of arrival
- AoD $\rightarrow$ Angle of departure

MIMO Model



Hybrid beamforming example with RF and digital steering

- The received signal at the UE can be expressed as

$$\mathbf{y} = \mathbf{W^H HFs} + \mathbf{W^H n}$$

- $\mathbf{y} \rightarrow$ Collected measurement
- $\mathbf{H} \in \mathcal{C}^{\mathbf{N}\times\mathbf{M}} \rightarrow$ Channel matrix
- $\mathbf{n} \rightarrow$ i.i.d $\mathcal{CN}$ noise
- $\mathbf{W} \rightarrow$ Combiner
- $\mathbf{F} \rightarrow$ Precoder
- $\mathbf{s} \rightarrow$ Known pilot symbol

Goal:

- Estimate **H** matrix ($N \times M$ elements) using **y** ; Underdetermined setting

Compressed Sensing Problem

Popular and Effective Approach:

- To pose the channel estimation problem as a sparse recovery problem with the help of **Beamspace** representation

Beamspace representation:

$$\mathbf{H} = \mathbf{A}_{\mathbf{UE,D}} \underbrace{\mathbf{H_V}}_{\text{Virtual matrix}} \mathbf{A}^{\mathbf{H}}_{\mathbf{BS,D}}$$

Array response at discretized grid points:

$$[\mathbf{A}_{\mathbf{BS,D}}]_{\mathbf{m},\mathbf{m}'} = \frac{1}{\sqrt{\mathbf{M}}} \exp \mathbf{j}2\pi(\mathbf{m}-1)\left(\frac{\mathbf{m}'-1}{\mathbf{M}} - \frac{1}{2}\right); \quad \mathbf{m},\mathbf{m}' \in [\mathbf{M}]$$

$$[\mathbf{A}_{\mathbf{UE,D}}]_{\mathbf{n},\mathbf{n}'} = \frac{1}{\sqrt{\mathbf{N}}} \exp \mathbf{j}2\pi(\mathbf{n}-1)\left(\frac{\mathbf{n}'-1}{\mathbf{N}} - \frac{1}{2}\right); \quad \mathbf{n},\mathbf{n}' \in [\mathbf{N}]$$

CS Problem:

$$\mathbf{y} = \mathbf{W^H A_{UE,D} H_V A^H_{BS,D} Fs} + \mathbf{W^H n}$$
$$\Downarrow$$
$$\mathbf{y} = \underbrace{(\mathbf{F^T} \otimes \mathbf{W^H})(\mathbf{A_{BS,D}}^* \otimes \mathbf{A_{UE,D}})}_{A} \underbrace{vec(\mathbf{H_V})}_{h_v} + \mathbf{n_W}$$
$$\Downarrow$$
$$\boxed{\mathbf{y} = \mathbf{Ah_V} + \mathbf{n_W}}$$

On-Grid and Off-Grid Effect

- **On-Grid:** MPCs AoA/AoD falls on the discretized spatial grid points
- **Off-Grid:** MPCs AoA/AoD falls off the discretized spatial grid points

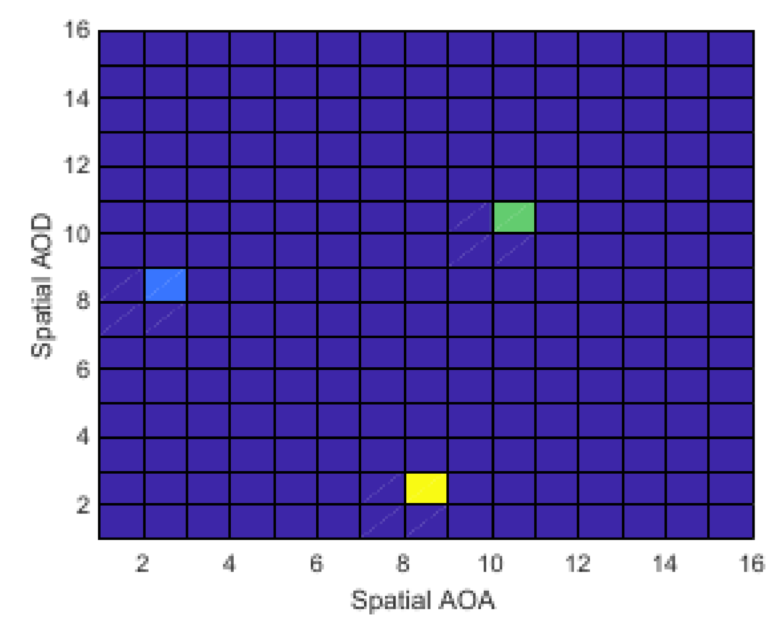


FIGURE – On-Grid Effect with 3 MPCs.

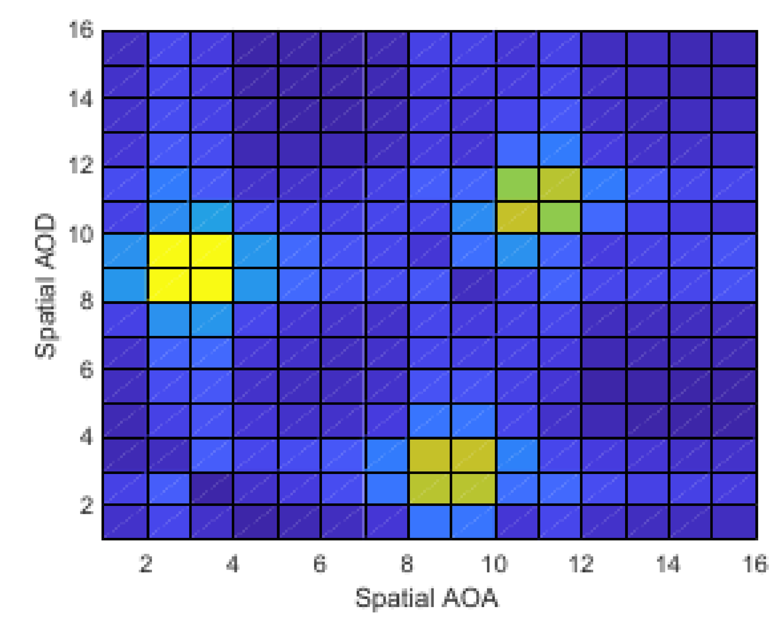


FIGURE – Off-Grid Effect with 3 MPCs.

- Off-Grid Effects ↓ → Number of discretized points ↑
- Increases computational complexity ↑
- Mutual coherence ↑

Our objective in a nutshell

- Design efficient CS algorithms to solve the MIMO channel estimation keeping off-grid effects in mind.

Dirichlet Structure in the Fourier Domain

Dirichlet Structure

- Each MPC → **Dirichlet kernel** in the continuum of the beamspace domain

Physical Model

$$\mathbf{H} = \alpha_l \mathbf{a}_{\mathbf{UE}}(\theta_l)\mathbf{a}_{\mathbf{BS}}{}^{\mathbf{H}}(\phi_l)$$

Virtual Model

$$[\mathbf{H_V}]_{\mathbf{m}',\mathbf{n}'} = \alpha_l \mathcal{D}(\varphi_l, \vartheta_l) \mathbf{e}^{-\mathbf{j}\pi(\vartheta_l(\mathbf{M}-1) - \varphi_l(\mathbf{N}-1))}$$

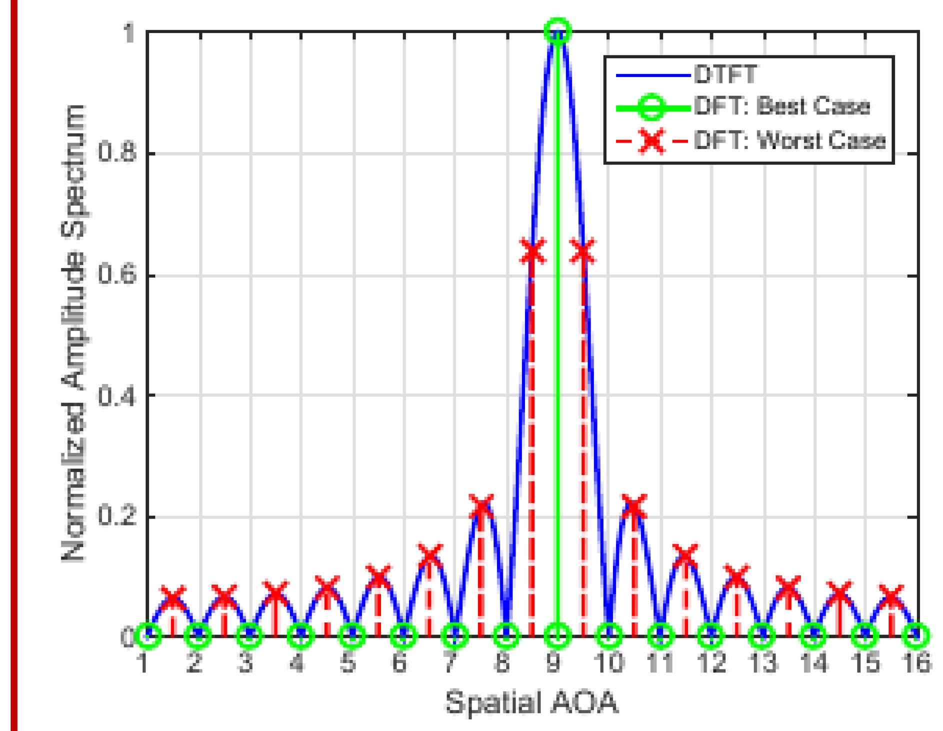
Dirichlet Kernel:

$$\mathcal{D}(\varphi_l,\vartheta_l) = \frac{1}{\sqrt{MN}} \frac{\sin(\pi\varphi_l M)}{\sin(\pi\varphi_l)} \frac{\sin(\pi\vartheta_l M)}{\sin(\pi\vartheta_l)}$$

Spatial AoD: $\varphi_l = \frac{n'-1}{N} - \frac{1}{2}\sin(\phi_l) - \frac{1}{2}$

Spatial AoA: $\vartheta_l = \frac{m'-1}{M} - \frac{1}{2}\sin(\theta_l) - \frac{1}{2}$

Amplitude Spectrum of Virtual Matrix



Spectrum of the virtual beamspace matrix in the spatial AOA domain with a single MPC with M=N= 16.

Key Idea:
The maxima of the Discrete Fourier Transform (DFT) may not correspond to the maxima of the discrete-time Fourier transform (DTFT) spectrum.

Goal:
Develop methods to find the maxima of the DTFT spectrum, and estimate DTFT peaks using dominant DFT points.

Proposed Solutions

- We employ OMP to obtain the dominant DFT points.
- **Method 1:** 2 projections and 2 least square (LS) steps .

Peak location and strength

$$m^* = m' + \frac{1}{2}\min\left\{\frac{[h_V]_{m'}}{[h_V]_{m'+1}}, \frac{[h_V]_{m'+1}}{[h_V]_{m'}}\right\}$$

$$\alpha_l^* = \frac{[h_V]_{m'}}{\mathcal{D}(\varphi_l)e^{-j\pi(\vartheta_l(M-1))}}$$

How do we obtain 2 dominant points ?

Projection: $j^* = \underset{j\in[N]}{\text{argmax}}\{|A^*(y - Ah_V^n)|_j\}$

LS Solution : $[h_V]_{m'} = A(:,j^*)^\dagger y$

- **Method 2:** 1 projection and 3 LS steps.

Peak location and strength

$$m^* = \frac{\tan(\frac{\pi}{M})}{\frac{\pi}{M}} \mathbb{R}\left(\frac{[h_V]_{m'-1}, -[h_V]_{m'+1}}{2[h_V]_{m'} - [h_V]_{m'-1} - [h_V]_{m'+1}}\right)$$

$$\alpha_l^* = \frac{[h_V]_{m'}}{\mathcal{D}(\varphi_l)e^{-j\pi\vartheta_l(M-1)}}$$

- $supp(h_v) = j \in [M] \rightarrow$ Index of the DFT points
- $[h_V]_{m'}\ \forall m' \in supp(h_v) \rightarrow$ Strength of the DFT points
- $m^* \rightarrow$ Estimated location of the DTFT peak
- $\alpha^* \rightarrow$ Estimated strength of the DTFT peak

Simulation Results

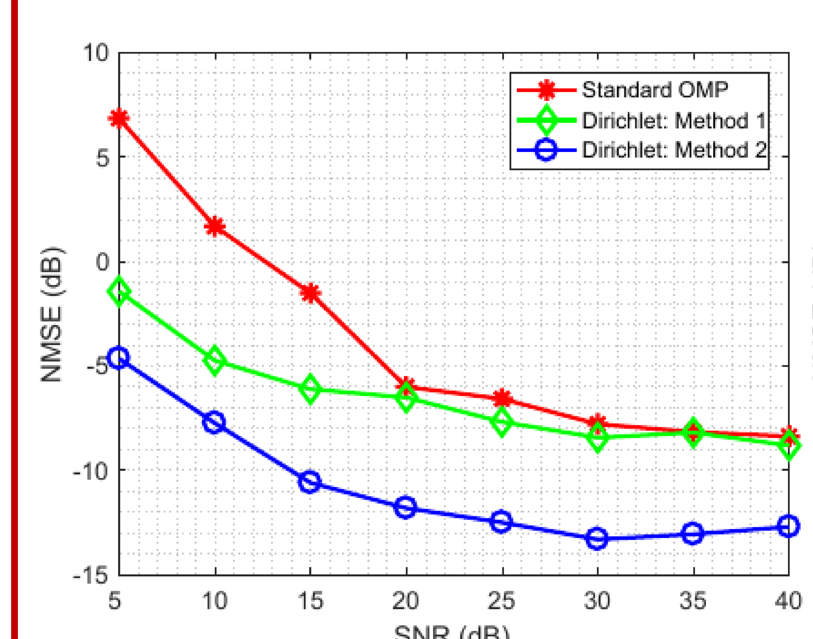


FIGURE – NMSE vs SNR in dB with number of measurement = $0.3N_aN_d$

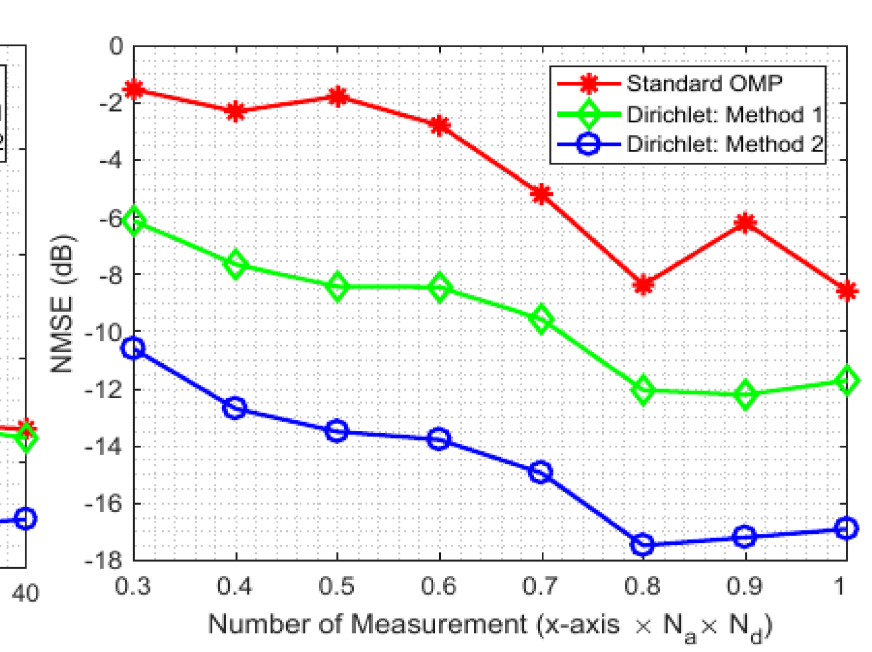


FIGURE – NMSE vs number of measurements with SNR = 15 dB.

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