Interference Modeling for 5G Networks with Millimeter Wave Beamforming

Hussain Elkotby and Mai Vu
Department of Electrical and Computer Engineering, Tufts University, USA

Tufts UNIVERSITY | School of Engineering

Introduction and Motivation

- millimeter wave (mmWave) is a promising solution to spectrum scarcity.
- Small mmWave cells lead to dense or extremely dense network deployments.
- Will mmWave network be noise or interference limited?
  - Need to characterize network interference.
- Factors affecting interference:
  - Density of interferers (users/base stations).
  - Transmission scheme.
  - Channel propagation model.

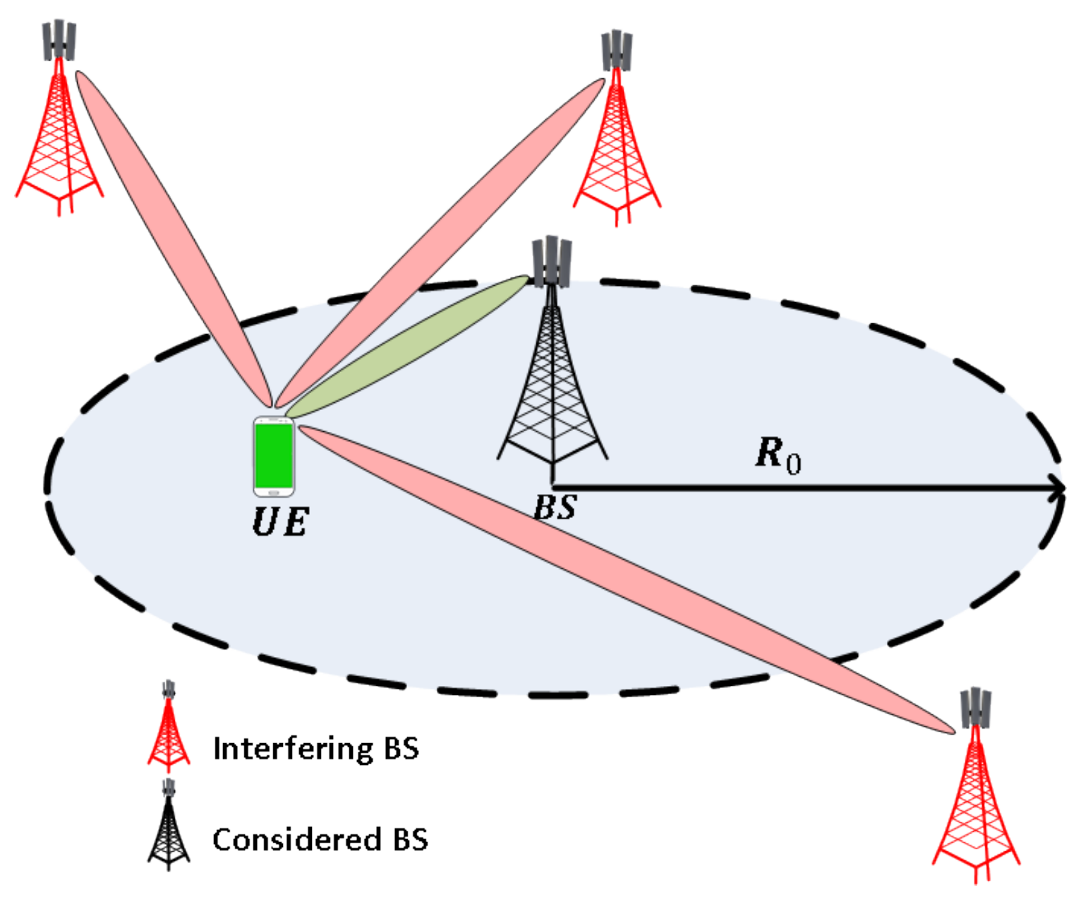

Fig. 1: System Model.

- Signal model:

$$\mathbf{y}_0 = \mathbf{H}_0\mathbf{w}x_0 + \mathbf{v}_0 + \mathbf{z}_0,$$

Network Performance

- Interference has a direct impact on system throughput and outage performance.
- Given fading and spatial realization, interference is modeled as $\mathcal{CN}(\mathbf{0}, \mathbf{Q}_0)$, where covariance matrix $\mathbf{Q}_0$ is random.
- Per-user capacity can then be written as:

$$C = \log\left(1 + \left\|\hat{\mathbf{h}}_0\right\|^2 P\right),$$

where $\quad \hat{\mathbf{h}}_0 = (\mathbf{R}_0)^{-1/2}\mathbf{H}_0\mathbf{w}_0 \quad$ and $\quad \mathbf{R}_0 = Q_0 + \sigma^2\mathbf{I}.$

- Due to channels fading and user locations randomness, outage probability at a target rate R_T is defined as:

$$p_o(R_T) = \mathbb{P}\{C < R_T\}$$

mmWave Channel Model

Narrowband channel propagation model:

$$\mathbf{H} = \sqrt{l(r)}\tilde{\mathbf{H}},$$

- Small-scale fading model $\tilde{\mathbf{H}}$:
  - Non-parametric: traditionally i.i.d. Rayleigh or Rician are considered.
  - Parametric:

$$\tilde{\mathbf{H}} = \frac{1}{\sqrt{N}}\sum_{k=1}^{K}\sum_{n=1}^{N} a_{k,n}\mathbf{u}_{rx}\left(\theta_{k,n}^{rx}, \phi_{k,n}^{rx}\right)\mathbf{u}_{tx}^{\dagger}\left(\theta_{k,n}^{tx}, \phi_{k,n}^{tx}\right)$$

- Large-scale fading model:

$$l(r) \;=\; \mathbb{B}\left(p(r)\right) L_l\beta_l r^{-\alpha_l} + \left(1 - \mathbb{B}\left(p(r)\right)\right) L_n\beta_n r^{-\alpha_n}$$

  - Probabilistic LOS and NLOS.
  - Shadowing is Lognormally distributed $L_i \sim \mathcal{N}(0, \sigma_i), \forall i \in \{l, n\}$.
  - Path loss model: floating intercept or close-in.
  - Probability that a link of length r is LOS:

$$p(r) = \left[\min\left(\frac{r_{BP}}{r}, 1\right)\left(1 - e^{-r/\alpha}\right) + e^{-r/\alpha}\right]^2$$

Interference Formulation

- The interference vector $\mathbf{v}_0$ is

$$\mathbf{v}_0 = \sum_{\mathbf{z}_k \in \Phi} \mathbf{H}_k\mathbf{w}_k x_k = \sum_{\mathbf{z}_k \in \Phi} \sqrt{P_k l\left(\|\mathbf{z}_k - \mathbf{t}_0\|_2\right)}\,\tilde{\mathbf{h}}_k U_k,$$

- The interference is $\mathcal{CN}(\mathbf{0}, \mathbf{Q}_0)$, where the covariance matrix $\mathbf{Q}_0$ is random with:
  - Diagonal elements as the interference power at each antenna element:

$$q_0 = \underbrace{\sum_{\mathbf{z}_k \in \Phi_l} l\left(\|\mathbf{z}_k - \mathbf{t}_0\|_2\right) g_k P_k}_{\text{LOS}} + \underbrace{\sum_{\mathbf{z}_k \in \Phi_n} l\left(\|\mathbf{z}_k - \mathbf{t}_0\|_2\right) g_k P_k}_{\text{NLOS}}.$$

  - Off-diagonal elements as the correlation.

Objective: To establish analytical distributions for the interference power and correlation.

Modeling Approaches

- Model the LOS interference power as Gamma:

$$\gamma_G(D) = \begin{cases} 0 & \text{with probability } p_0(D); \\ \tilde{\gamma}_G(D) & \text{with probability } 1 - p_0(D), \end{cases}$$

- $p_0(D)$: probability that LOS interference is 0.
- Param. est. via moment matching (MM) or MLE.
- Model the NLOS interference power as a mixture:

$$f_Y(y|\tilde{\theta}) \;=\; w_1 f_{\gamma_{IG}}(y|\lambda) + w_2 f_{\gamma_{IW}}(y|c),$$

- Param. est. using a combination of MM and MLE.

Initial Results for i.i.d. Channels

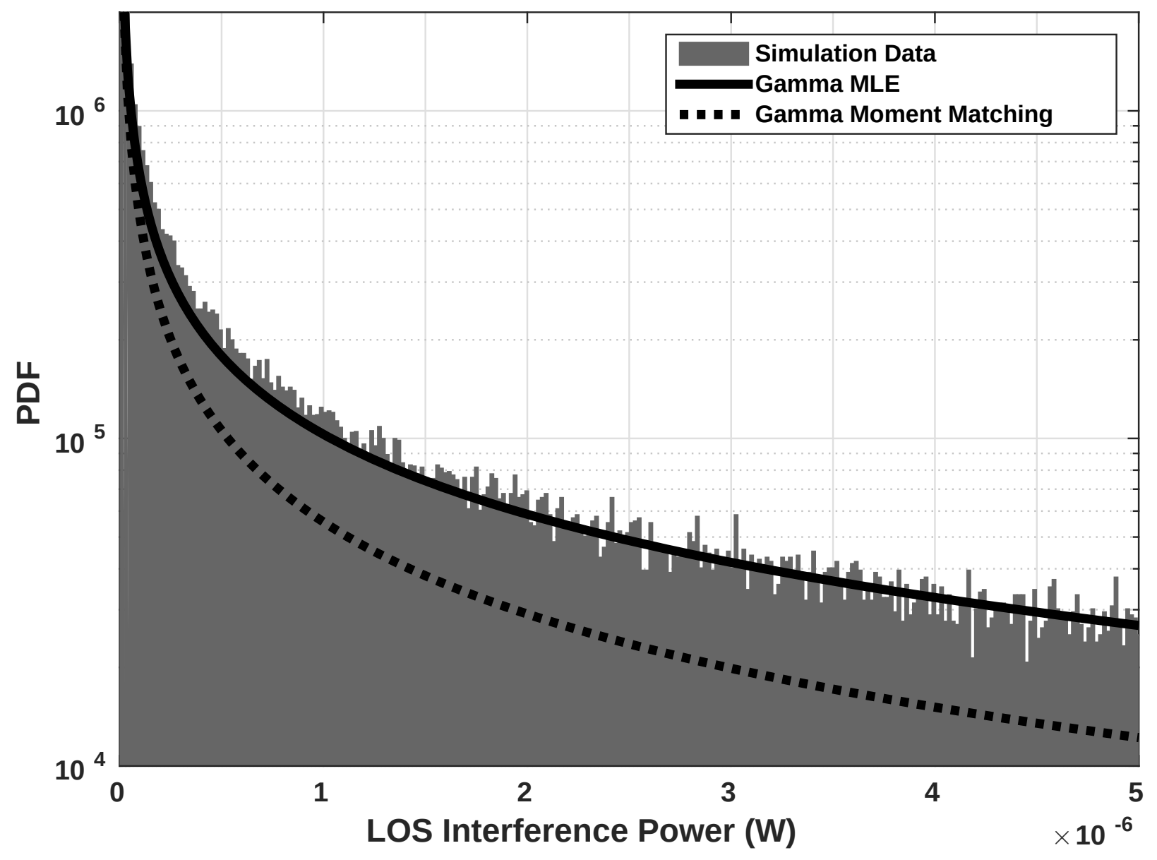

Fig. 2: Sample PDF of LOS interference models at $\alpha_l = 3.5$, $\sigma_l = 4$ dB, ($P_{max} = 30$ dB, $D = 75$ m).

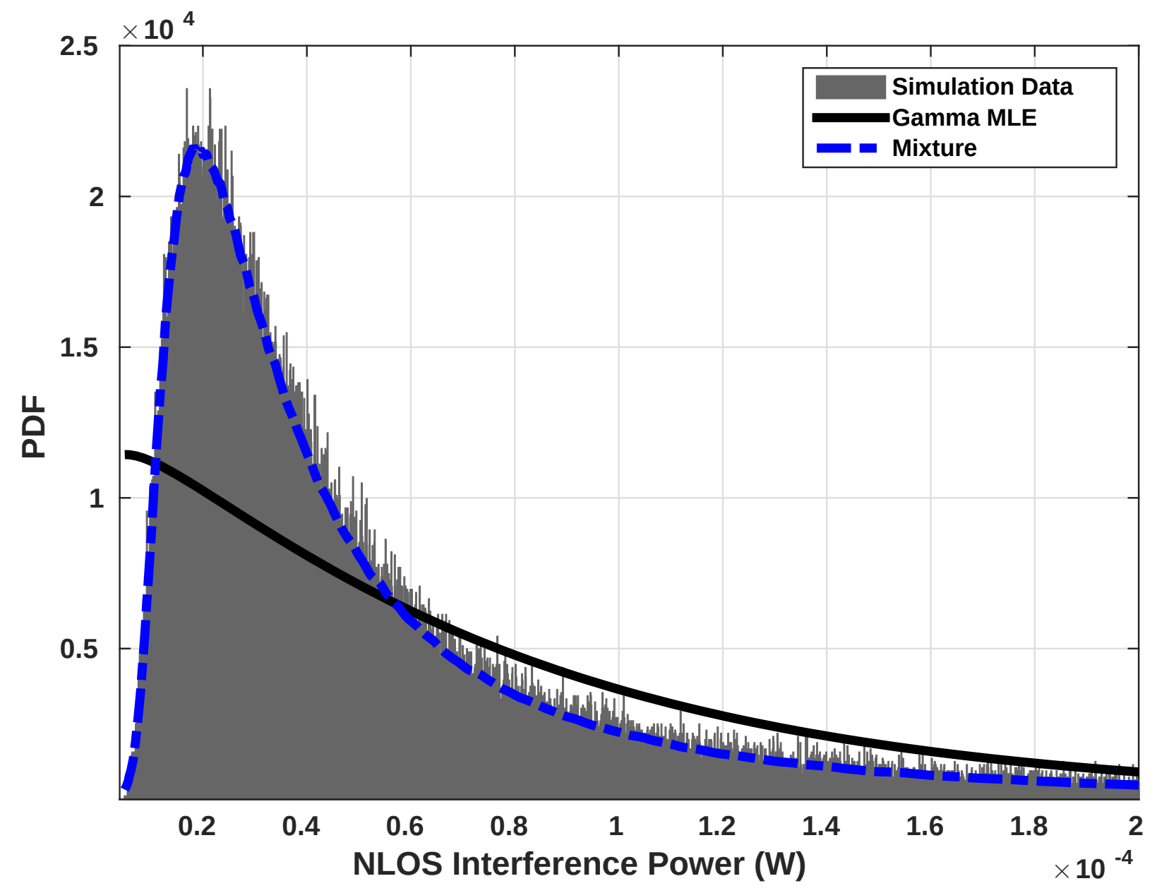

Fig. 3: Sample PDF of NLOS interference models at $\alpha_l = 3.5$, $\sigma_l = 4$ dB, ($P_{max} = 30$ dB, $D = 75$ m).

Future Work

- Model interference under the mmWave parametric channel model.
- Fit interference model parameters as functions of mmWave propagation characteristics.
- Model the interference correlation.
- Examine application of interference models in mmWave network analysis and evaluation.